Basic Differentiation

i. Compute $\frac{dy}{dx}$ for each of the following expressions:

- $y = x^2$
- $y = x^5$
- $y = 3x^3$
- $y = 4x^2 + 3x$
- $y = 5x^{-4}$
- $y = 3x^6 + 9x^{-3}$
- $y = \sqrt{x}$
- $y = 6\sqrt[3]{x}$
- $y = \frac{5}{2\sqrt{x}}$
- $y = x^4 + 6\sqrt{x}$
- $y = x^{\frac{3}{2}}$

ii. Calculate the derivative $\left(f'(x)\right)$ for each of the following functions:

- $f(x) = 3$
- $f(x) = -x$
- $f(x) = \frac{1}{x^2}$
- $f(x) = 6x^{10}$
- $f(x) = \frac{5}{x^2 + x}$
- $f(x) = \frac{3}{4}x^{-\frac{1}{2}}$

iii. A water tank is filled with water at a rate depending on time. The volume of water, in litres, being added at a time t seconds, is given by:

$$v(t) = 4t^2 + 2, \qquad 0 \leq t \leq 10$$

- What is the rate of change in volume of water being added to the tank?
- Find the rate of change of v when $t = 5$
- What is the rate of change of v initially?

iv. Compute the derivative of each of the following functions:

- $f(x) = (2x + 5)(x - 1)$
- $g(x) = \frac{x^2 + 5x - 3}{x}$
- $h(x) = \frac{3x^4 + 9x - 7x^2}{x^2}$
- $y(x) = \frac{(x + 4)^2}{x}$