Basic Algebra

Basic Algebraic Identities

$$x + x = 2x \qquad x^p \times x^q = x^{p+q}$$
$$x \times x = x^2 \qquad (x^p)^q = x^{pq}$$

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i. Solve the equation:

$$\frac{5}{r} + 3 = 2$$

Solution: We want to unwrap the r term so we reach an answer in the form $r = ?$, so:

$\frac{5}{r} + 3 = 2$	`(Original equation)`
$5 + 3r = 2r$	`(Multiply each term by` r`)`
$5 = -r$	`(Subtract` $3r$ `from both sides)`
$r = -5$	`(Divide both sides by` 4`)`

We can substitute our value back into the original equation to check our answer:

$$\begin{aligned} \frac{5}{-5} + 3 &= 2 \\ -1 + 3 &= 2 \\ 2 &= 2 \end{aligned}$$

ii. Solve the equation:

$$9y + 3 = 5y + 13$$

Solution: We wish to have terms with y's only on one side of the equation, so:

$9y + 3 = 5y + 13$	`(Original equation)`
$4y + 3 = 13$	`(Subtract` $5y$ `from both sides)`
$4y = 10$	`(Subtract` 3 `from both sides)`
$y = {}^{10}/_{4}$	`(Divide both sides by` 4`)`
$= 2.5$	

iii. Solve the equation:

$$\frac{6}{f + 2} = \frac{2}{f - 1}$$

Solution: We want to get rid of the two fractions and have terms with f's only on one side of the equation, so:

$$\begin{aligned}
\frac{6}{f+2} &= \frac{2}{f-1} && \texttt{(Original equation)}\\
6 &= \frac{2}{f-1}\cdot(f+2) && \texttt{(Multiply both sides by } (f+2)\texttt{)}\\
6\cdot(f-1) &= 2\cdot(f+2) && \texttt{(Multiply both sides by } (f-1)\texttt{)}\\
6f-6 &= 2f+4 && \texttt{(Multiply out the brackets)}\\
4f-6 &= 4 && \texttt{(Subtract } 2f \texttt{ from both sides)}\\
4f &= 10 && \texttt{(Add } 6 \texttt{ to both sides)}\\
f &= {}^{10}/_{4} = 2.5 && \texttt{(Divide both sides by } 4\texttt{)}
\end{aligned}$$

We can substitute our value back into the original equation to check our answer:

$$\begin{aligned}
\frac{6}{2.5+2} &= \frac{2}{2.5-1}\\
\frac{6}{4.5} &= \frac{2}{1.5}\\
\frac{12}{9} &= \frac{4}{3}\\
\frac{4}{3} &= \frac{4}{3}
\end{aligned}$$

iv. **Factorise:**

$$5x^2+15x$$

Solution: We want to simplify the equation by finding any common factors of each of the terms. We note that both terms of the equation have a common factor of 5 and x. We take those factors outside the bracket and divide each term by the factor as follows:

$$\begin{aligned}
5x^2+15x &= 5x\left(\frac{5x^2+15x}{5x}\right) && \texttt{(Take } 5x \texttt{ out of the bracket and divide by the factor)}\\
&= 5x\left(\frac{5x^2}{5x}+\frac{15x}{5x}\right) && \texttt{(Split the division into two divisions)}\\
&= 5x\,(x+3) && \texttt{(Compute the division)}
\end{aligned}$$

v. **Expand the equation:**

$$10y(y+4+y^2)$$

Solution: We want to multiply each of the terms in the bracket by the factor outside the bracket:

$$\begin{aligned}
10y(y+4+y^2) &= (10y)y+(10y)4+(10y)y^2 && \texttt{(Multiply each term inside the bracket by } 10y\texttt{)}\\
&= 10y^2+40y+10y^3 && \texttt{(Compute the multiplication)}
\end{aligned}$$

vi. Rearrange the equation to make d the subject:

$$r = \frac{1}{2}(c - d)$$

Solution: We wish to have d on the left-hand side so we must 'unwrap' the right-hand side as follows:

$$\begin{aligned} r &= \frac{1}{2}(c-d) && \texttt{(Original equation)} \\ 2r &= c - d && \texttt{(Multiply both sides by 2)} \\ 2r - c &= -d && \texttt{(Subtract } c \texttt{ from both sides)} \\ c - 2r &= d && \texttt{(Multiply both sides by } -1\texttt{)} \\ d &= c - 2r && \texttt{(Swap the two sides)} \end{aligned}$$

vii. Rearrange the equation below to make e the subject

$$f = 10 - \frac{\sqrt{e}}{5}$$

Solution: We wish to have e on the left-hand side so we must 'unwrap' the right-hand side as follows:

$$\begin{aligned} f &= 10 - \frac{\sqrt{e}}{5} && \texttt{(Original equation)} \\ f - 10 &= -\frac{\sqrt{e}}{5} && \texttt{(Subtract 10 from both sides)} \\ 10 - f &= \frac{\sqrt{e}}{5} && \texttt{(Multiply both sides by } -1\texttt{)} \\ 5(10 - f) &= \sqrt{e} && \texttt{(Multiply both sides by 5)} \\ (5(10 - f))^2 &= e && \texttt{(Square both sides)} \\ e &= 25(10 - f)^2 && \texttt{(Swap the sides and simplify)} \end{aligned}$$