Sum of Arithmetic Series

For a given sequence u_k, with first term a, common difference d and k^{th} term:

$$u_k = a + (k-1)d$$

the sum of the first n terms is given by:

$$S_n = \sum_{k=1}^{n} u_k = \sum_{k=1}^{n} \big(a + (k-1)d\big) = a + (a+d) + (a+2d) + \cdots + \big(a + (n-1)d\big)$$

$$= \frac{1}{2} n \big(2a + (n-1)d\big)$$

Proof:

We have:

$$\begin{array}{cccccccccc}
 & S_n & = & a & + & a+d & + & a+2d & + \cdots + & a+(n-1)d \\
+ & S_n & = & a+(n-1)d & + & a+(n-2)d & + & a+(n-3)d & + \cdots + & \mathrm{a} \\
\hline
 & 2S_n & = & 2a+(n-1)d & + & 2a+(n-1)d & + & 2a+(n-1)d & + \cdots + & 2a+(n-1)d
\end{array}$$

$$2S_n = n\big(2a + (n-1)d\big)$$

$$S_n = \frac{1}{2} n \big(2a + (n-1)d\big)$$