Derivative of a quadratic polynomial (from first principles)

Below we show from first principles that the derivative of a quadratic polynomial $f(x) = ax^2 + bx + c$ is given

$$f'(x) = 2ax + b.$$

Proof

From the first principle definition of a derivative, we have that for a continuous function $f(x)$, it's derivative $f'(x)$ is given by:

$$f'(x) = \lim_{\triangle x \to 0} \frac{f(x + \triangle x) - f(x)}{\triangle x}$$

So, for the quadratic polynomial $f(x) = ax^2 + bx + c$, we have:

$$\begin{aligned}
f'(x) &= \lim_{\triangle x \to 0} \frac{a(x + \triangle x)^2 + b(x + \triangle x) + c - (ax^2 + bx + c)}{\triangle x} \\
&= \lim_{\triangle x \to 0} \frac{ax^2 + 2ax\triangle x + a(\triangle x)^2 + bx + b\triangle x + c - ax^2 - bx - c}{\triangle x} \\
&= \lim_{\triangle x \to 0} \frac{2ax\triangle x + a(\triangle x)^2 + b\triangle x}{\triangle x} \\
&= \lim_{\triangle x \to 0} 2ax + a\triangle x + b \\
&= 2ax + b
\end{aligned}$$

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