

# Unit C1 Core Mathematics 1

AS compulsory unit for GCE AS and GCE Mathematics, GCE AS and GCE Pure Mathematics

## C1.1 Unit description

Algebra and functions; coordinate geometry in the  $(x, y)$  plane; sequences and series; differentiation; integration.

## C1.2 Assessment information

### Preamble

Construction and presentation of rigorous mathematical arguments through appropriate use of precise statements and logical deduction, involving correct use of symbols and appropriate connecting language is required. Students are expected to exhibit correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient', and notation such as  $\therefore$ ,  $\Rightarrow$ ,  $\Leftarrow$  and  $\Leftrightarrow$ .

### Examination

The examination will consist of one 1½ hour paper. It will contain about ten questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

For this unit, students may **not** have access to any calculating aids, including log tables and slide rules.

### Formulae

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

#### Quadratic equations

$$ax^2 + bx + c = 0 \text{ has roots } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Differentiation**

function

$x^n$

derivative

$nx^{n-1}$

**Integration**

function

$x^n$

integral

$$\frac{1}{n+1} x^{n+1} + c, n \neq -1$$

**1 Algebra and functions****What students need to learn:**

Laws of indices for all rational exponents.

The equivalence of  $a^{m/n}$  and  $\sqrt[n]{a^m}$  should be known.

Use and manipulation of surds.

Students should be able to rationalise denominators.

Quadratic functions and their graphs.

The discriminant of a quadratic function.

Completing the square. Solution of quadratic equations.

Solution of quadratic equations by factorisation, use of the formula and completing the square.

Simultaneous equations: analytical solution by substitution.

For example, where one equation is linear and one equation is quadratic.

Solution of linear and quadratic inequalities.

For example,  $ax + b > cx + d$ ,  
 $px^2 + qx + r \geq 0$ ,  $px^2 + qx + r < ax + b$ .

Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation.

Students should be able to use brackets. Factorisation of polynomials of degree  $n$ ,  $n \leq 3$ , eg  $x^3 + 4x^2 + 3x$ . The notation  $f(x)$  may be used. (Use of the factor theorem is *not* required.)

Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.

Functions to include simple cubic functions and the reciprocal function  $y = \frac{k}{x}$  with  $x \neq 0$ .

Knowledge of the term asymptote is expected.

Knowledge of the effect of simple transformations on the graph of  $y = f(x)$  as represented by  $y = af(x)$ ,  $y = f(x) + a$ ,  $y = f(x + a)$ ,  $y = f(ax)$ .

Students should be able to apply one of these transformations to any of the above functions (quadratics, cubics, reciprocal) and sketch the resulting graph.

Given the graph of any function  $y = f(x)$  students should be able to sketch the graph resulting from one of these transformations.

## 2 Coordinate geometry in the $(x, y)$ plane

### What students need to learn:

Equation of a straight line, including the forms  $y - y_1 = m(x - x_1)$  and  $ax + by + c = 0$ .

To include:

- (i) the equation of a line through two given points
- (ii) the equation of a line parallel (or perpendicular) to a given line through a given point. For example, the line perpendicular to the line  $3x + 4y = 18$  through the point  $(2, 3)$  has equation  $y - 3 = \frac{4}{3}(x - 2)$ .

Conditions for two straight lines to be parallel or perpendicular to each other.

## 3 Sequences and series

### What students need to learn:

Sequences, including those given by a formula for the  $n$ th term and those generated by a simple relation of the form  $x_{n+1} = f(x_n)$ .

Arithmetic series, including the formula for the sum of the first  $n$  natural numbers.

The general term and the sum to  $n$  terms of the series are required. The proof of the sum formula should be known.

Understanding of  $\Sigma$  notation will be expected.

## 4 Differentiation

### What students need to learn:

The derivative of  $f(x)$  as the gradient of the tangent to the graph of  $y = f(x)$  at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.

Differentiation of  $x^n$ , and related sums and differences.

Applications of differentiation to gradients, tangents and normals.

For example, knowledge that  $\frac{dy}{dx}$  is the rate of change of  $y$  with respect to  $x$ . Knowledge of the chain rule is not required.

The notation  $f'(x)$  may be used.

For example, for  $n \neq 1$ , the ability to differentiate expressions such as  $(2x + 5)(x - 1)$  and  $\frac{x^2 + 5x - 3}{3x^{1/2}}$  is expected.

Use of differentiation to find equations of tangents and normals at specific points on a curve.

## 5 Integration

### What students need to learn:

Indefinite integration as the reverse of differentiation.

Integration of  $x^n$ .

Students should know that a constant of integration is required.

For example, the ability to integrate expressions such as

$$\frac{1}{2}x^2 - 3x^{-\frac{1}{2}} \text{ and } \frac{(x+2)^2}{x^{\frac{1}{2}}} \text{ is}$$

expected.

Given  $f'(x)$  and a point on the curve, students should be able to find an equation of the curve in the form  $y = f(x)$ .