C1 Sample Exam Paper

Time Allowed: 1 hour 30 minutes

1. Differentiate, with respect to x:

(a) $y = x^4 + \dfrac{3}{x} + 1$ **[3]**

(b) $y = (2 + \sqrt{x})^2$ **[3]**

2. (a) Rationalise the denominator of the fraction $\dfrac{5}{\sqrt{3}}$. **[1]**

(b) Show that the expression: **[4]**

$$\frac{5 - \sqrt{3}}{6 + \sqrt{3}}$$

can be written in the form: $\dfrac{a + b\sqrt{3}}{3}$, where a and b are integers to be found.

3. (a) Show that the function: **[3]**

$$f(x) = \frac{(x+2)^2(x-1)}{x}, \ x \neq 0$$

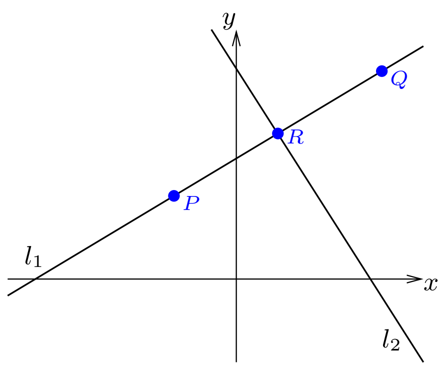
can be written as

$$f(x) = x^2 + 3x - 4x^{-1}.$$

(b) Find $f'(x)$. **[3]**

(c) Hence, find the equation of the tangent to curve $y = f(x)$ at $x = 1$. **[5]**

4. The diagram below shows the lines l_1 and l_2 in the (x, y) plane. The line l_1 passes through the points P, Q, R and is perpendicular to l_2.



(a) Given that $P = (-1, 4)$ and $Q = (7, 5)$, find an equation for l_1, giving your answer in the form $ay + bx + c = 0$, where a, b, c are integers. **[4]**

(b) Verify that the point $R = (3, 9/2)$ is the mid-point of PQ. **[2]**

(c) Given that l_2 passes through R, find an equation for l_2. **[5]**

5. List all the possible integer values of x for which the following inequality holds: **[5]**

$$x^2 + 2x - 10 \leq 3x + 10$$

6. An arithmetic sequence a_n has first term $a_1 = 87$ and subsequent terms have a common difference of -4.

(a) What is the 7^{th} term in the sequence? **[2]**

(b) What is the highest value of n for which a_n is positive? **[4]**

(c) Hence, find the maximum value of S_n, the sum of the first n terms of the sequence. **[4]**

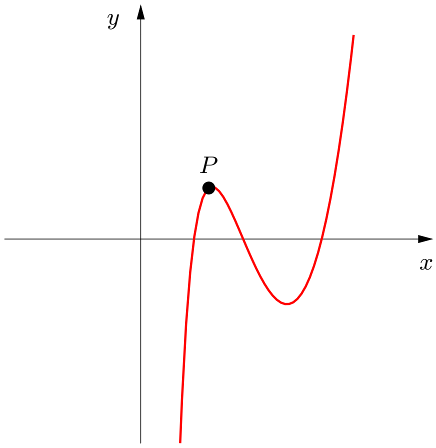
7. Solve the simultaneous equations, leaving your answers in surd form: **[12]**

$$y = 2x^2 - 3x + 5$$
$$y + 2x - 7 = 0$$

8. The curve C has equation $y = f(x),\ x > 0$, with $f'(x) = 3x^2 + \dfrac{8}{x^3} - 10\sqrt{x}$.

(a) Given that the curve C passes through the point $P(1, 1)$, find $f(x)$. **[6]**

The graph below shows a plot of the curve C.



The curve C is transformed by $y = f(x) + k$, with k a positive integer. The co-ordinates of P is now $(0, 1)$.

(b) What is the value of k? **[1]**

Without specific calculation, sketch the following curves and give the co-ordinates of P in each case.

i. $y = f(x + 3)$ **[3]**

ii. $y = 2f(x)$ **[3]**

9. The equation $(\frac{1}{4}k)x^2 - 10x + 4k = x - 2k$ has two real solutions for x.

(a) Show that k satisfies $k^2 < 22$. **[5]**

(b) Hence, find all the possible values of k, such that the condition holds. **[3]**

10. A sequence is defined recursively by:

$$\begin{cases} x_3 = 5 \\ x_{n+1} = \dfrac{x_n + p}{2} \end{cases}$$

with p a non-zero constant.

(a) Write down an expression in p for x_4. **[1]**

(b) Show that $x_1 = 20 - 3p$. **[4]**

Given that $x_4 = 4$,

(c) Find the value of x_1. **[2]**

11. Given that $f(x) = x^2 - 8x + 14,\ x \geq 0$,

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where a, b are intergers. **[3]**

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q. **[5]**

The line y = 21 meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + \sqrt{q}$, where p and q are integers. **[4]**

