Boolean Identities

Recall: $\text{NOT}(A) = \overline{A}$, $\text{AND}(A, B) = A \cdot B$, $\text{OR}(A, B) = A + B$.

Basic Identities

Given an input X, the following identities hold:

- $X \cdot X = X$
- $X \cdot \overline{X} = \texttt{false}$
- $X + X = X$
- $X + \overline{X} = \texttt{true}$
- $X \cdot \texttt{true} = X$
- $X \cdot \texttt{false} = \texttt{false}$
- $X + \texttt{true} = \texttt{true}$
- $X + \texttt{false} = X$

Commutative Properties

Given two inputs X and Y, we can interchange X and Y in some cases:

- $X \cdot Y = Y \cdot X$
- $X + Y = Y + X$

De Morgan's Laws

Given two inputs X and Y, the following identities hold:

- $\overline{X \cdot Y} = \overline{X} + \overline{Y}$ *or equivalently* $\text{NAND}(X, Y) = \text{OR}(\text{NOT}(X), \text{NOT}(Y))$
- $\overline{X + Y} = \overline{X} \cdot \overline{Y}$ *or equivalently* $\text{NOR}(X, Y) = \text{AND}(\text{NOT}(X), \text{NOT}(Y))$

To remember De Morgan's Laws, use the phrase " break the line, change the sign".