The Discriminant

i. **Compute the value of the discriminant of each of the following quadratic functions and state how many real solutions (roots) the equation will have:**

- $a(x) = x^2 + 10x + 4$

  Solution: $\Delta = 10^2 - 4 \times 1 \times 4 = 84$
  Two roots

- $b(x) = x^2 - 5x + 9$

  Solution: $\Delta = (-5)^2 - 4 \times 1 \times 9 = -11$
  No roots

- $c(x) = 2x^2 + 3x + 10$

  Solution: $\Delta = 3^2 - 4 \times 2 \times 10 = -71$
  No roots

- $d(x) = (x + 2)^2$

  Solution: $\Delta = 4^2 - 4 \times 1 \times 4 = 0$
  One root

- $e(x) = 5 - x^2 + 9x$

  Solution: $\Delta = 9^2 - 4 \times -1 \times 5 = 29$
  Two roots

- $f(x) = 4x^2 - 5x$

  Solution: $\Delta = (-5)^2 - 4 \times 4 \times 0 = 25$
  Two roots

- $g(x) = 4 - 6x + x^2$

  Solution: $\Delta = (-6)^2 - 4 \times 1 \times 4 = 20$
  Two roots

- $h(x) = -1/2x^2 + 3x - 6$

  Solution: $\Delta = 3^2 - 4 \times -1/2 \times -6 = -3$
  No roots

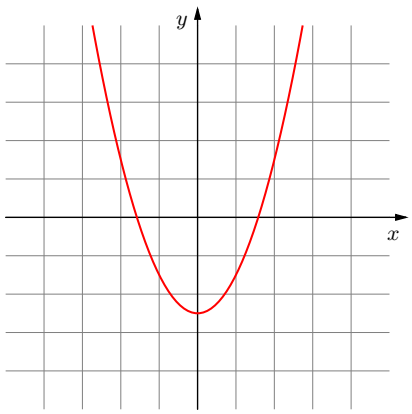
- $i(x) = (x + 3)(x - 5)$

  Solution: $\Delta = (-2)^2 - 4 \times 1 \times -15 = 64$
  Two roots

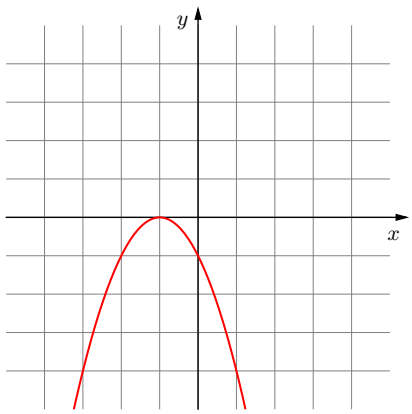
- $j(x) = x(x + 2)$

  Solution: $\Delta = 2^2 - 4 \times 1 \times 0 = 4$
  Two roots

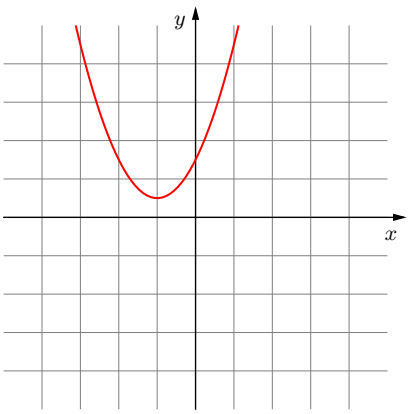
ii. **State whether the discriminant of the following functions will be positive, negative or zero:**



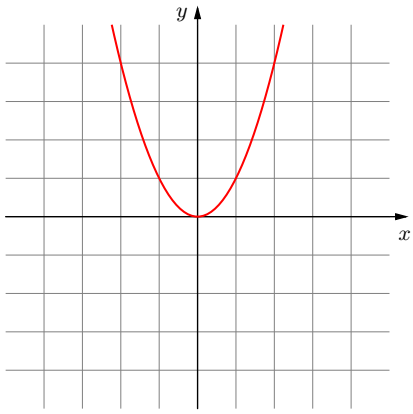
Positive



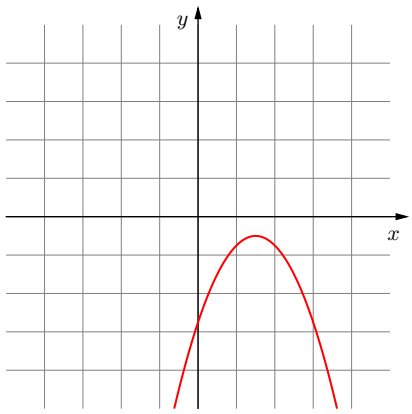
Zero



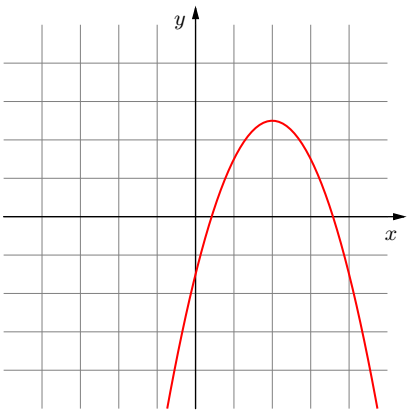
Negative



Zero



Negative



Positive

iii. **The equation $2x^2 - 3x - (k+1) = 0$, where k is constant, has no real roots. Find the set of possible values of k.**

Solution: $(-3)^2 - 4 \times 2 \times -(k+1) < 0 \implies 9 + 8(k+1) < 0 \implies 8k + 17 < 0$
$\implies 8k < -17 \implies k < -17/8$

iv. **The equation $x^2 + 2px + (3p+4) = 0$, where p is a positive constant, has equal roots. Find the value of p.**

Solution: $(2p)^2 - 4 \times 1 \times (3p+4) = 0 \implies 4p^2 - 12p - 16 = 0$
$\implies p^2 - 3p - 4 = 0 \implies (p+1)(p-4) = 0 \implies p = 4$ or -1.
$p > 0 \implies p = 4$

