

Basic Algebra

Basic Algebraic Identities

$$\begin{aligned}x + x &= 2x \\x \times x &= x^2\end{aligned}$$

$$\begin{aligned}x^p \times x^q &= x^{p+q} \\(x^p)^q &= x^{pq}\end{aligned}$$

i. Solve the equation:

$$9y + 3 = 5y + 13$$

Solution: We wish to have terms with y 's only on one side of the equation, so:

$$\begin{aligned}9y + 3 &= 5y + 13 && \text{(Original equation)} \\4y + 3 &= 13 && \text{(Subtract } 5y \text{ from both sides)} \\4y &= 10 && \text{(Subtract 3 from both sides)} \\y &= \frac{10}{4} && \text{(Divide both sides by 4)} \\&= 2.5\end{aligned}$$

ii. Factorise:

$$5x^2 + 15x$$

Solution: We want to simplify the equation by finding any common factors of each of the terms. We note that both terms of the equation have a common factor of 5 and x . We take those factors outside the bracket and divide each term by the factor as follows:

$$\begin{aligned}5x^2 + 15x &= 5x \left(\frac{5x^2 + 15x}{5x} \right) && \text{(Take } 5x \text{ out of the bracket and divide by the factor)} \\&= 5x \left(\frac{5x^2}{5x} + \frac{15x}{5x} \right) && \text{(Split the division into two divisions)} \\&= 5x(x + 3) && \text{(Compute the division)}\end{aligned}$$

iii. Expand the equation:

$$10y(y + 4 + y^2)$$

Solution: We want to multiply each of the terms in the bracket by the factor outside the bracket:

$$\begin{aligned}10y(y + 4 + y^2) &= (10y)y + (10y)4 + (10y)y^2 && \text{(Multiply each term inside the bracket by } 10y\text{)} \\&= 10y^2 + 40y + 10y^3 && \text{(Compute the multiplication)}\end{aligned}$$

iv. Rearrange the equation to make d the subject:

$$r = \frac{1}{2}(c - d)$$

Solution: We wish to have d on the left-hand side so we must 'unwrap' the right-hand side as follows:

$$\begin{aligned}r &= \frac{1}{2}(c - d) && \text{(Original equation)} \\2r &= c - d && \text{(Multiply both sides by 2)} \\2r - c &= -d && \text{(Subtract } c \text{ from both sides)} \\c - 2r &= d && \text{(Multiply both sides by } -1\text{)} \\d &= c - 2r && \text{(Swap the two sides)}\end{aligned}$$

v. Rearrange the equation below to make e the subject

$$f = 10 - \frac{\sqrt{e}}{5}$$

Solution: We wish to have e on the left-hand side so we must 'unwrap' the right-hand side as follows:

$$\begin{aligned}f &= 10 - \frac{\sqrt{e}}{5} && \text{(Original equation)} \\f - 10 &= -\frac{\sqrt{e}}{5} && \text{(Subtract 10 from both sides)} \\10 - f &= \frac{\sqrt{e}}{5} && \text{(Multiply both sides by } -1\text{)} \\5(10 - f) &= \sqrt{e} && \text{(Multiply both sides by 5)} \\(5(10 - f))^2 &= e && \text{(Square both sides)} \\e &= 25(10 - f)^2 && \text{(Swap the sides and simplify)}\end{aligned}$$